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The University of Warwick
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THE COMPUTATION OF GREEN FUNCTIONS OF
FINITE CHEVALLEY GROUPS OF TYPE
 E_n ($n = 6, 7, 8$)

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July 1982

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CONTENTSPAGE

1.	The abstract computation	1
2.	The computer calculations	8
	Acknowledgements	16
	References	16
3.	The tables	
	I Springer's representations	17
	E_6	27-37
	E_7	39-65
	E_8	67-165
	II Inclusions and branching of closures of unipotent classes	167
	E_8	169
	E_7	172

THE COMPUTATION OF GREEN FUNCTIONS OF FINITE CHEVALLEY

GROUPS OF TYPE E_n ($n = 6, 7, 8$)

W.M. Beynon and N. Spaltenstein

The main purpose of this report is to present tables of Green functions for the finite groups E_n ($n = 6, 7, 8$). It is intended to complement our paper [BS], in which the mathematical significance and interpretation of these results is discussed in detail. A brief account of the computation is also included; some of the technical details may be of interest to others wishing to develop similar programs.

1. The abstract computation

To put the computation in its proper context, and to understand its justification, the reader should be familiar with the first two sections of [BS]. For our purpose, it will suffice to give a sketchy resume of the notation and formulae required to formulate the computational problem and its solution.

Let k be an algebraic closure of a finite field F_q ($q = p^e$, p a prime). Let G be a simple algebraic group of type E_n ($n = 6, 7$ or 8) over k defined over F_q with

Frobenius endomorphism $F : G \rightarrow G$. Let W be the Weyl group of G , and U the unipotent variety of G . As in [BS], if H is a finite group, then \hat{H} will be used to denote the set of isomorphism classes of irreducible complex representations of H .

As explained in [BS], the Green functions to be computed can be thought of as integer-valued functions on W indexed by elements u in U^F . Each such function Q_u^G depends only on the G^F -conjugacy class of u in U^F . We initially assume that each Q_u^G can be expressed in the form:

$$Q_u^G = \sum_{\kappa} \left(\sum_i n_{u,i,\kappa} q^i \right) \kappa \quad (1.1)$$

where the coefficients $n_{u,i,\kappa}$ are integers, and the summation is over κ in \hat{W} . Our calculations vindicate this assumption, and explicitly determine the coefficients $n_{u,i,\kappa}$ in the class of formulae (1.1), or equivalently, the family of polynomials $\sum_i n_{u,i,\kappa} q^i$ indexed by elements κ of \hat{W} for a representative u of each G^F -conjugacy class of U^F . The families Q_1^G have already been computed (see [BL]).

The principle of the computation is to determine the Q_u^G 's in a particular sequence

$$Q_{u_0}^G, Q_{u_1}^G, Q_{u_2}^G, \dots, Q_{u_{\text{NOCL}-1}}^G, \quad (1.2)$$

(where $(u_0 = 1, u_1, u_2, \dots, u_{\text{NOCL}-1})$ is a transversal of the G^F -conjugacy classes of U^F) in such a way that at each step

$$Q_{u_{i+1}}^G, Q_{u_{i+2}}^G, \dots, Q_{u_{i+s}}^G \quad (1.3)$$

where $s = 1, 2, 3$ or (in one exceptional step, for G of type E_8) 7 , are determined from $Q_{u_j}^G$'s with $j \leq i$. As explained in detail in [BS], when $s \geq 1$, the elements $u = u_{i+1}, u_{i+2}, \dots, u_{i+s}$ are chosen to form a set of representatives for G^F -orbits within C^F , where C is a unipotent class, and are in 1-1 correspondence with conjugacy classes in a group $A(u)$ of isomorphism type (1) ($s=1$), C_2 ($s=2$), S_3 ($s=3$) or S_5 ($s=7$). The sequence (1.2) then corresponds (in the obvious fashion) to an ordering of the unipotent classes

$$C_1, C_2, \dots, C_k$$

with the property that $\dim C_i \leq \dim C_j$, if $i \leq j$. This ensures that if C_i is contained in the closure of C_j , then $i \leq j$. (The sequences of the form (1.2) used in our computations for G of type E_6, E_7 and E_8 are described explicitly below.)

It remains to give details of the way in which the polynomial families (1.3) are computed at each step.

Suppose then that $C, u = u_{i+1}, u_{i+2}, \dots, u_{i+s}$ and $A(u)$ are as above. If $s = 1, 2$ or 3 then to each ϕ in $A(u)^\wedge$ there corresponds a unique element of W^\wedge , denoted $\rho_{u,\phi}$. Adapting formula (2.2) in [BS].

$$|G^F| \prod_{r=1}^s |C_G(u_{i+r})^F|^{-1} \langle Q_{u_{i+r}}^G, \rho_{u,\phi} \rangle_W Q_{u_{i+r}}^G \quad (1.4)$$

$$= q^N \epsilon_{\rho_{u,\phi}} Q_1^G - |G^F| \prod_{j=1}^i |C_G(u_j)^F|^{-1} \langle Q_{u_j}^G, \rho_{u,\phi} \rangle_W Q_{u_j}^G$$

for each ϕ in $A(u)^\wedge$, where $N = N(G)$ is the number of positive roots of G , and $|G^F|$ and $|C_G(u_j)^F|$ are known polynomials in q . Suppose now that $a_1 = 1, a_2, \dots, a_s$ are representatives of conjugacy classes of $A(u)$ associated with $u_{i+1}, u_{i+2}, \dots, u_{i+s}$ respectively, under the 1-1 correspondence mentioned above. For the computation, we assume that for a suitable choice of u in C^F :

$$\langle Q_{u_{i+r}}^G, \rho_{u,\phi} \rangle = q^{\beta(u)} \phi(a_r) \text{ for } 1 \leq r \leq s, \quad (1.5)$$

where $\beta(u)$ is known. (This assumption is invalid if G is twisted of type E_6 , and for one particular class C

in E_8 if $q \equiv 2 \pmod{3}$. In both cases, the required results can still be obtained by interpreting the computed results appropriately (see [BS] and section 3 below for details.) We can then combine (1.4) and (1.5) to obtain

$$q^{\beta(u)} |G^F| \sum_{r=1}^s |C_G(u_{i+r})^F|^{-1} \phi(a_r) Q_{u_{i+r}}^G \quad (1.6)$$

$$= q^N \epsilon_{\rho_{u,\phi}} Q_1^G - |G^F| \sum_{j=0}^i |C_G(u_j)^F|^{-1} \langle Q_{u_j}^G, \rho_{u,\phi} \rangle_W Q_{u_j}^G$$

for each ϕ in $A(u)^\wedge$. This is a system of linear equations with as many equations as unknowns which has the $Q_{u_{i+r}}^G$ ($1 \leq r \leq s$) as unique solution.

In the exceptional case (which arises once only, when G is of type E_8), there is a unipotent class C' , together with representatives

$$v = u_{t+1}, \dots, u_{t+7} \quad (1.7)$$

for the G^F -orbits of $(C')^F$, with $A(v)$ isomorphic to S_5 .

In this case, $\rho_{v,\phi}$ is defined for all but one element of $A(v)^\wedge$, and the analogue of (1.6) supplies 6 equations in 7 unknowns. The exceptional element of $A(v)^\wedge$ corresponds to the sign representation ϵ of S_5 .

For b in $A(v)$, let v_b denote the representative associated with the conjugacy class of b of $A(v)$ in (1.7). Let

$$Q_{v,\varepsilon}^G = |A(v)|^{-1} \sum_{b \in A(v)} \varepsilon(b) Q_{v_b}^G.$$

It can be shown [BS] that the polynomial

$$|C_G(v_b)^F| = |C_{A(v)}(b)| q^{40}, \quad (1.8)$$

where $C_{A(v)}(b)$ is the centraliser of b in $A(v)$. From (1.8), it is easy to see that if $Q_{v,\varepsilon}^G = 0$, then

$$|G^F| \sum_{r=1}^s |C_G(u_{i+r})^F|^{-1} \varepsilon(a_r) Q_{u_{i+r}}^G = 0. \quad (1.9)$$

As explained in [BS], it is possible to assume that $Q_{v,\varepsilon}^G = 0$ and carry out the computation of (possibly spurious) families $Q_{u_{t+r}}^*$ ($1 \leq r \leq 7$) and $Q_{u_j}^*$ ($j > t+7$) by adjoining (1.9) to the 6 equations derived as in (1.6).

To verify that the results obtained in this way are correct, it then suffices to check that if $u = u_{j+1}, \dots, u_{j+s}$ ($j \geq t+7$) as in (1.3) are associated with a subsequent step, then (letting $Q_{u_i}^*$ denote $Q_{u_i}^G$ for $0 \leq i \leq t$):

$$|G^F| \sum_{i=0}^{j+s} |C_G(u_i)^F|^{-1} \langle Q_{u_i}^*, \rho_{u, \phi} \rangle^2 = q^N \langle \epsilon \rho_{u, \phi}, Q_{1, \rho_{u, \phi}}^G \rangle \quad (1.10)$$

for all ϕ in $A(u)^\wedge$.

Our main computation, which is described in fuller detail below, calculates the Green functions Q_u^G as in (1.2) by the above method. However, if $s > 1$, and

$$u = u_{i+1}, u_{i+2}, \dots, u_{i+s}$$

are as in (1.3), the results presented in our tables are the polynomial families $Q_{u, \phi}^G$ ($\phi \in A(u)^\wedge$), where

$$Q_{u, \phi}^G = \sum_{r=1}^s |C_{A(u)}(a_r)|^{-1} \phi(a_r) Q_{u_{i+r}}^G, \quad (1.11)$$

in the notation of (1.6). (In particular, the fact that

$Q_{v, \epsilon} = 0$ when G is of type E_8 and $A(v) \cong S_5$ is explicit in the tables.)

2. The computer calculations

In this section, we give brief technical details of the actual computations made. Since the essential features of the computation are illustrated adequately when G is of type E_6 , we consider this case in detail. The special abstract and practical problems associated with the case when G is of type E_8 will be briefly discussed later.

Assume that G is of type E_n , where $n = 6$, and that W denotes the Weyl group of G . For convenience, the elements of W^\wedge will be referred to as "characters", and the G^F -conjugacy classes of U^F as "classes".

The data initially available is as listed below.

The sequence

$$1 = u_0, u_1, \dots, u_{\text{NOCL}-1} \quad (\text{NOCL} = 25) \quad (2.1)$$

of representatives of classes associated with (1.2).

The vector

$$\beta(u_0), \beta(u_1), \dots, \beta(u_{\text{NOCL}-1}) \quad (2.2)$$

where $N = \beta(1) \quad (= 36)$.

The family of polynomials in $Z[q]$:

$$|C_G(u_i)^F| \quad (0 \leq i < \text{NOCL})$$

where $|G^F| = |C_G(1)^F|$, and in general $|C_G(u_i)^F|$ has degree $2\beta(i) + n = 2\beta(i) + 6$.

The set of characters

$$\kappa_0, \kappa_1, \dots, \kappa_{\text{NOCH}-1} \quad (\text{NOCH} = 25) \quad (2.4)$$

which are identified with the columns of the 25×25 character table of W .

The vector

$$c_0, c_1, \dots, c_{\text{NOCH}-1} \quad (2.5)$$

specifying the sizes of conjugacy classes in W .

The family Q_1^G of polynomials in $Z[q]$:

$$\sum_{k=0}^N n_{1,k,\kappa} q^k \quad \text{for } \kappa \text{ in } W^\wedge \quad (2.6)$$

For each pair u and ϕ in $A(u)^\wedge$ as in (1.4) the associated character $\rho_{u,\phi}$ is also known. As described previously, if $u = u_{i+1}, u_{i+2}, \dots, u_{i+s}$ (where $s = 1, 2$ or 3) are the representatives associated with one step of the computation as in (1.3), then the u_i 's are in 1-1 correspondence with the conjugacy classes of $A(u)$, and hence in (non-canonical) 1-1 correspondence with the elements of $A(u)^\wedge$. It is computationally convenient to exploit the artificial correspondence

$$u_{i+r} \leftrightarrow \phi_r \in A(u)^\wedge \quad (1 \leq r \leq s)$$

set up in this way, and to consider ρ_{u, ϕ_r} as corresponding to u_{i+r} . The family ρ_{u, ϕ_r} can then be conceived as a vector of characters

$$\rho(u_0) = \kappa_\sigma(0), \rho(u_1) = \kappa_\sigma(1), \dots, \rho(u_{\text{NOCL}-1}) = \kappa_\sigma(\text{NOCL}-1) \quad (2.7)$$

where $\rho(u_{i+r})$ is defined as ρ_{u, ϕ_r} . (The map σ is in fact a permutation of $(0, 1, \dots, \text{NOCL}-1 = \text{NOCH}-1)$.)

The result required (c.f. (1.1)) is the indexed set of families of polynomials

$$Q_{u_i}^G \quad (0 \leq i < \text{NOCL})$$

where $Q_{u_i}^G$ comprises polynomials

$$z_{i\kappa}(q) = \sum_{k=0}^{\beta(i)} n_{u_i, k, \kappa} q^k \text{ for } \kappa \text{ in } W^\wedge, \quad (2.8)$$

and Q_1^G is given as input (2.6).

As explained above, the families $Q_{u_i}^G$ are computed in sequence from (1.6). There are two computational steps for which $s = 2$; the families $(Q_{u_5}^G, Q_{u_6}^G)$ and $(Q_{u_{21}}^G, Q_{u_{22}}^G)$

are computed together. There is one step for which $s = 3$; the families $(Q_{u_{13}}^G, Q_{u_{14}}^G, Q_{u_{15}}^G)$ are computed together.

For $0 \leq i < \text{NOCL}$, let $f_i(q)$ denote $|G^F||C_G(u_i)^F|^{-1}$.

For each ϕ_t in $A(u)^\wedge$ ($1 \leq t \leq s$), equation (1.6) can then be expressed more simply as

$$\begin{aligned} q^{\beta(u)} \sum_{r=1}^s \phi_t(a_r) f_{i+r}(q) Q_{u_{i+r}}^G \\ = q^N \varepsilon \kappa_\sigma(t) Q_1^G - \sum_{j=0}^i \langle Q_{u_j}^G, \kappa_\sigma(t) \rangle_W f_j(q) Q_{u_j}^G. \end{aligned} \quad (2.9)$$

Since we work with this variant of (1.6), the polynomials $f_i(q)$ are pre-computed from (2.3). Moreover, in view of the form of (2.9), it proves to be useful to store the families

$$f_i(q) Q_{u_i}^G \quad (0 \leq i < \text{NOCL}) \quad (2.10)$$

together with the families $Q_{u_i}^G$. (It is in fact necessary to work with an integer multiple of the f_i 's to avoid rational arithmetic.)

To further simplify (2.9), observe that

$$\varepsilon \kappa_\sigma(t) Q_1^G = \sum_{\tilde{\kappa}} \left(\sum_{k=0}^N n_{1,k,\tilde{\kappa}} \tilde{\kappa}^k \right) \varepsilon \kappa_\sigma(t)^{\tilde{\kappa}}. \quad (2.11)$$

Using (2.4) and (2.5) the product of characters $\epsilon_{\kappa_{\sigma(t)} \tilde{\kappa}}$ can be expressed in the form

$$\sum_{\kappa} e_{\tilde{\kappa}, \kappa} \text{ for } \tilde{\kappa} \in \hat{W}. \quad (2.12)$$

Substituting in (2.10) and re-arranging gives

$$q^N \epsilon_{\kappa_{\sigma(t)}} = \sum_{\kappa} \left(\sum_{k=0}^N \left(\sum_{\tilde{\kappa}} n_{1,k,\tilde{\kappa}} e_{\tilde{\kappa}, \kappa} \right) q^k \right) \kappa \quad (2.13)$$

Using (1.1) and (2.13) it is then possible to re-write (2.9) as a family of equations indexed by characters, viz:

$$\sum_{r=1}^S \phi_t(a_r) f_{i+r}(q) z_{i+r\kappa}(q) \quad (2.14)$$

$$= q^{N-\beta(u)} \sum_{k=0}^N \left(\sum_{\tilde{\kappa}} n_{1,k,\tilde{\kappa}} e_{\tilde{\kappa}, \kappa} \right) q^k - \sum_{j=0}^i z_{j\kappa_{\sigma(t)}}(q) f_j(q) z_{j\kappa}(q)$$

for κ in \hat{W} .

The system of equations (2.14), where ϕ_t ranges over $A(u)^{\wedge}$, is the explicit version of (1.6) used by our programs. Note that for each character κ , the determination of the polynomials $f_{i+r}(q) z_{i+r\kappa}(q)$ is straightforward, since the coefficients on the LHS of (2.14) depend only upon $A(u)$. The polynomials $z_{i+r\kappa}(q)$ are then found by polynomial division.

It remains to consider the way in which polynomial data is represented in the above computation. In the case of E_6 , it would be possible to represent all polynomials simply by sufficiently long coefficient vectors. The families $Q_{u_i}^G$ might then be stored in coefficient form in a $25 \times 25 \times 37$ array of integers, and the families (2.10) similarly in a $25 \times 25 \times 73$ array. Such representations are too wasteful to be feasible for E_7 and E_8 , and an alternative method is used. Almost all polynomials are stored as triples (U, L, A) , where U and L are non-negative integers, and A is a coefficient vector of length $U-L+1$. Thus (U, L, A) encodes the polynomial

$$q^L (a_0 + a_1 q + \dots + a_{U-L} q^{U-L})$$

where $A = (a_0, a_1, \dots, a_{U-L})$ and $a_0 a_{U-L} \neq 0$. Multiplication and division of polynomials is performed relative to this representation. Coefficient arrays are used to represent polynomials only in the computation of the RHS of (2.14); for this purpose, it is convenient to have Q_1^G in array form so that the computation of the first term is a matrix multiplication, and the accumulation of terms is also most simply carried out on a coefficient array.

When G is of type E_7 , the method of computation described above applies subject to minor changes. For E_7 , the parameters NOCH, NOCL and N are respectively 60, 60 and 63.

When G is of type E_8 , there are significant differences. In this case, the number of classes (NOCL) is 113, the number of characters (NOCH) is 112, and $N = 120$. For the particular sequence (2.1) used in our program, the exceptional step occurs when the families

$$(Q_{u_{59}}^G, Q_{u_{60}}^G, Q_{u_{61}}^G, Q_{u_{62}}^G, Q_{u_{63}}^G, Q_{u_{64}}^G, Q_{u_{65}}^G)$$

are computed together that is, $t = 58$ and $v = u_{59}$ in the context of (1.7). Here $\rho_{v,\varepsilon}$ is undefined, and the vector (2.7) accordingly has a missing entry at this point. The system of equations

$$\sum_{r=1}^7 \varepsilon(a_r) f_{58+r}(q) z_{58+r\kappa}(q) = 0 \text{ for } \kappa \text{ in } \hat{W}, \quad (2.15)$$

is used in conjunction with (2.14) in this exceptional step. The computation is otherwise similar to that carried out for E_6 and E_7 . (The required checks are easily made subject to storing the data needed to verify (1.10) as it is computed.)

The computation of our tables for E_8 also presented peculiar technical difficulties, largely on account of the enormous amount of data involved. (The input data required by this program comprised about 36,000 integers.) Compiling the data was itself a major part of the programming effort, for which a number of auxiliary programs were written. Most of the computation was programmed in C for the local VAX-11/750 system, but the pre-computation of the polynomials $f_i(q)$ for E_8 (and E_7) was carried out on an APL system at IBM Warwick.

In the main computation for G of type E_8 , a very large proportion of the processing time was spent in computing the 112 matrices $(e_{\tilde{k},k})$ of size 112×112 , which required double-length (64 bit 2's complement) arithmetic. The computation of the first 52 tables required approximately 800 minutes process time. To cope with errors in data, it was (inevitably) necessary to devise a recovery version of the program to read the tables so far computed, and re-start the computation. An early version of this program required a notional 600 minutes process time to recover the first 52 tables! This problem was the result of using an inappropriate storage allocation mechanism, and modification reduced the elapsed time required for recovery of data from days to minutes. This was particularly significant since

recovery of all 113 tables was needed to cast the results into the form (1.11) in which they appear below.

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- [BS] Green functions of finite Chevalley groups of type E_n ($n=6,7,8$), W.M. Beynon and N. Spaltenstein (1982).

3. The tables

Tables for Springer's representations are given below. The Green functions can be expressed in terms of these. For E_7 and E_8 , the part which is relevant to applications (E) and (F) of [BS], that is, the determination of all inclusions of closures of unipotent classes and the branching of such closures, is given in a separate table.

The characteristic is assumed to be good. For Green functions we must further assume that p and q are large.

I. Springer's representations.

We give the integers $n_{u,\phi,i,\chi}$ ($u \in G$ unipotent, $\phi \in A(u)^\wedge$, $i \in \mathbb{N}$, $\chi \in W^\wedge$) defined by $n_{u,\phi,i,\chi} = \langle \chi \otimes \phi, H^{2i}(B_u^G) \rangle_{W \times A(u)}$ (see [BS]). We adopt the convention that the representation in degree 0 is trivial.

The Green functions can be recovered as follows. Suppose first that G is split and let C be a unipotent class in G . There are then two cases to consider:

(i) there is a split element u_1 in C^F (in the sense of [BS]). If the G^F -class of $u \in C^F$ corresponds to $a \in A(u)$ and if T_w is an F -stable maximal torus corresponding to $w \in W$, then

$$Q_{T_w, G}(u) = Q_u^G(w) = \sum_{\substack{\chi \in W^{\wedge} \\ \phi \in A(u)^{\wedge} \\ i \geq 0}} n_{u, \phi, i, \chi} \phi(a) \chi(w) q^i$$

(ii) there is no split element in C^F ; this is the case only if G is of type E_8 , C is the class $D_8(a_3)$ and $q \equiv -1 \pmod{3}$.

In this case we choose $u \in C^F$ in such a way that F acts trivially on $A(u) \cong S_3$. With the same notations as above, we have then

$$Q_{T_w, G}(u) = \sum_{\substack{\chi \in W^{\wedge} \\ i \geq 0}} (n_{u, 1, i, \chi} + \sigma(a)n_{u, \sigma, i, \chi} - \epsilon(a)n_{u, \epsilon, i, \chi}) \chi(w) q^i$$

where σ (resp. ϵ) is the two-dimensional (resp. the sign) representation of S_3 .

If G is twisted of type E_6 , the Green functions are obtained by replacing q by $-q$ and w by $w_0 w$ in the polynomials giving the Green functions for split groups of type E_6 . Here w_0 is the longest element of W . The details are given in [BS].

There is one table for each $\chi \in W^\wedge$. We use Frame's notations for χ . The lines in the tables correspond to the integer i . The columns correspond to the pairs (u, ϕ) ($u \in G$ unipotent, $\phi \in A(u)^\wedge$) taken up to conjugacy.

The following facts have been used to condense the tables for E_7 and E_8 :

- (a) for χ and (u, ϕ) fixed, the integers i such that $n_{u, \phi, i, \chi} \neq 0$ all have the same parity (this is an empirical observation).
- (b) in most cases the column giving $n_{1, 1, i, \chi}$ (χ fixed) is palindromic, that is, there is an integer m such that $n_{1, 1, i, \chi} = n_{1, 1, m-i, \chi}$ for all i . In some cases we have truncated this column, and indicated the integer $m/2$ (m is even) by a sign \succ .

We have numbered the conjugacy classes of pairs (u, ϕ) . For the class of u we use the same notations as Bala and Carter. The characters of $A(u) \cong S_r$ are labelled by partitions of r , with (r) for the trivial character and $(1, \dots, 1)$ for the sign. The numbering is as follows.

E₆

number	u	ϕ	number	u	ϕ
0	\emptyset	(1)	13	$D_4(a_1)$	(2,1)
1	A_1	(1)	14	$D_4(a_1)$	(1,1,1)
2	$2A_1$	(1)	15	A_4	(1)
3	$3A_1$	(1)	16	D_4	(1)
4	A_2	(2)	17	A_4+A_1	(1)
5	A_2	(1,1)	18	$D_5(a_1)$	(1)
6	A_2+A_1	(1)	19	A_5	(1)
7	$2A_2$	(1)	20	$E_6(a_3)$	(2)
8	A_2+2A_1	(1)	21	$E_6(a_3)$	(1,1)
9	A_3	(1)	22	D_5	(1)
10	$2A_2+A_1$	(1)	23	$E_6(a_1)$	(1)
11	$D_4(a_1)$	(3)	24	E_6	(1)
12	$D_4(a_1)$	(3)			

The class $E_6(a_3)$ is denoted A_5+A_1 by Mizuno.

E_7	number	u	ϕ	number	u	ϕ
	0	\emptyset	(1)	20	A_3+2A_1	(1)
	1	A_1	(1)	21	D_4	(1)
	2	$2A_1$	(1)	22	$D_4(a_1)+A_1$	(2)
	3	$(3A_1)''$	(1)	23	$D_4(a_1)+A_1$	(1)
	4	$(3A_1)'$	(1)	24	A_3+A_2	(2)
	5	A_2	(2)	25	A_3+A_2	(1,1)
	6	A_2	(1,1)	26	$A_3+A_2+A_1$	(1)
	7	$4A_1$	(1)	27	A_4	(2)
	8	A_2+A_1	(2)	28	A_4	(1,1)
	9	A_2+A_1	(1,1)	29	D_4+A_1	(1)
	10	A_2+2A_1	(1)	30	$(A_5)''$	(1)
	11	A_2+3A_1	(1)	31	A_4+A_1	(2)
	12	$2A_2$	(1)	32	A_4+A_1	(1,1)
	13	A_3	(1)	33	A_4+A_2	(1)
	14	$(A_3+A_1)''$	(1)	34	$D_5(a_1)$	(2)
	15	$2A_2+A_1$	(1)	35	$D_5(a_1)$	(1,1)
	16	$(A_3+A_1)'$	(1)	36	$D_5(a_1)+A_1$	(1)
	17	$D_4(A_1)$	(3)	37	$(A_5)'$	(1)
	18	$D_4(a_1)$	(2,1)	38	A_5+A_1	(1)
	19	$D_4(a_1)$	(1,1,1)			

<u>number</u>	<u>u</u>	<u>ϕ</u>	<u>number</u>	<u>u</u>	<u>ϕ</u>
39	$E_6(a_3)$	(2)	50	$E_7(a_4)$	(1,1)
40	$E_6(a_3)$	(1,1)	51	$E_6(a_1)$	(2)
41	$D_6(a_2)$	(1)	52	$E_6(a_1)$	(1,1)
42	$E_7(a_5)$	(3)	53	D_6	(1)
43	$E_7(a_5)$	(2,1)	54	$E_7(a_3)$	(2)
44	$E_7(a_5)$	(1,1,1)	55	$E_7(a_3)$	(1,1)
45	D_5	(1)	56	E_6	(1)
46	A_6	(1)	57	$E_7(a_2)$	(1)
47	D_5+A_1	(1)	58	$E_7(a_1)$	(1)
48	$D_6(a_1)$	(1)	59	E_7	(1)
49	$E_7(a_4)$	(2)			

The correspondence with Mizuno's notations is as follows:

Bala-Carter	Mizuno	Bala-Carter	Mizuno
$E_7(a_3)$	D_6+A_1	$E_6(a_3)$	$(A_5+A_1)'$
$E_7(a_4)$	$D_6(a_1)+A_1$	A_5+A_1	$(A_5+A_1)''$
$E_7(a_5)$	$D_6(a_2)+A_1$		

E_8					
number	u	ϕ	number	u	ϕ
0	\emptyset	(1)	18	$D_4(a_1)$	(1,1,1)
1	A_1	(1)	19	$2A_2+2A_1$	(1)
2	$2A_1$	(1)	20	D_4	(1)
3	$3A_1$	(1)	21	$D_4(a_1)+A_1$	(3)
4	A_2	(2)	23	$D_4(a_1)+A_1$	(2,1)
5	A_2	(1,1)	24	$D_4(a_1)+A_1$	(1,1,1)
6	$4A_1$	(1)	25	A_3+A_2	(2)
7	A_2+A_1	(2)	26	A_3+A_2	(1,1)
8	A_2+A_1	(1,1)	27	A_4	(2)
9	A_2+2A_1	(1)	28	A_4	(1,1)
10	A_3	(1)	29	$A_3+A_2+A_1$	(1)
11	A_2+3A_1	(1)	30	$D_4(a_1)+A_2$	(2)
12	$2A_2$	(2)	31	$D_4(a_1)+A_2$	(1,1)
13	$2A_2$	(1,1)	32	D_4+A_1	(1)
14	$2A_2+A_1$	(1)	33	$2A_3$	(1)
15	A_3+A_1	(1)	34	A_4+A_1	(2)
16	$D_4(a_1)$	(3)	35	A_4+A_1	(1,1)
17	$D_4(a_1)$	(2,1)	36	$D_5(a_1)$	(2)

number	u	ϕ	number	u	ϕ
37	$D_5(a_1)$	(1,1)	57	$E_7(a_5)$	(2,1)
38	A_4+2A_1	(2)	58	$E_7(a_5)$	(1,1,1)
39	A_4+2A_1	(1,1)	59	$E_8(a_7)$	(5)
40	A_4+A_2	(1)	60	$E_8(a_7)$	(4,1)
41	$A_4+A_2+A_1$	(1)	61	$E_8(a_7)$	(3,2)
42	$D_5(a_1)+A_1$	(1)	62	$E_8(a_7)$	(3,1,1)
43	A_5	(1)	63	$E_8(a_7)$	(2,2,1)
44	D_4+A_2	(2)	64	$E_8(a_7)$	(2,1,1,1)
45	D_4+A_2	(1,1)	65	$E_8(a_7)$	(1,1,1,1,1)
46	$E_6(a_3)$	(2)	66	D_5+A_1	(1)
47	$E_6(a_3)$	(1,1)	67	$D_6(a_1)$	(2)
48	D_5	(1)	68	$D_6(a_1)$	(1,1)
49	A_4+A_3	(1)	69	A_6	(1)
50	$D_5(a_1)+A_2$	(1)	70	A_6+A_1	(1)
51	A_5+A_1	(1)	71	$E_7(a_4)$	(2)
52	$E_6(a_3)+A_1$	(2)	72	$E_7(a_4)$	(1,1)
53	$E_6(a_3)+A_1$	(1,1)	73	D_5+A_2	(2)
54	$D_6(a_2)$	(2)	74	D_5+A_2	(1,1)
55	$D_6(a_2)$	(1,1)	75	$E_6(a_1)$	(2)
56	$E_7(a_5)$	(3)	76	$E_6(a_1)$	(1,1)

number	u	ϕ	number	u	ϕ
77	D_6	(1)	97	$E_8(b_5)$	(2,1)
78	$D_7(a_2)$	(2)	98	$E_8(b_5)$	(1,1,1)
79	$D_7(a_2)$	(1,1)	99	D_7	(1)
80	E_6	(1)	100	$E_8(a_5)$	(2)
81	A_7	(1)	101	$E_8(a_5)$	(1,1)
82	$E_6(a_1)+A_1$	(2)	102	$E_7(a_1)$	(1)
83	$E_6(a_1)+A_1$	(1,1)	103	$E_8(b_4)$	(2)
84	$E_7(a_3)$	(2)	104	$E_8(b_4)$	(1,1)
85	$E_7(a_3)$	(1,1)	105	$E_8(a_4)$	(2)
86	$E_8(b_6)$	(3)	106	$E_8(a_4)$	(1,1)
87	$E_8(b_6)$	(2,1)	107	E_7	(1)
88	$E_8(b_6)$	(1,1,1)	108	$E_8(a_3)$	(2)
89	$D_7(a_1)$	(2)	109	$E_8(a_3)$	(1,1)
90	$D_7(a_1)$	(1,1)	110	$E_8(a_2)$	(1)
91	E_6+A_1	(1)	111	$E_8(a_1)$	(1)
92	$E_8(a_6)$	(3)	112	E_8	(1)
93	$E_8(a_6)$	(2,1)			
94	$E_8(a_6)$	(1,1,1)			
95	$E_7(a_2)$	(1)			
96	$E_8(b_5)$	(3)			

The correspondence with Mizuno's notations is as

follows:

Bala-Carter	Mizuno	Bala-Carter	Mizuno
$E_8(a_3)$	E_7+A_1	$E_8(a_7)$	$2A_4$
$E_8(a_4)$	D_8	$E_7(a_3)$	D_6+A_1
$E_8(b_4)$	$E_7(a_1)+A_1$	$E_7(a_4)$	$D_6(a_1)+A_1$
$E_8(a_5)$	$D_8(a_1)$	$E_7(a_5)$	A_5+A_2
$E_8(a_5)$	$D_8(a_1)$	$E_7(a_5)$	A_5+A_2
$E_8(b_5)$	$E_7(a_2)+A_1$	$E_6(a_3)+A_1$	A_5+2A_1
$E_8(a_6)$	A_8	$E_6(a_3)$	$(A_5+A_1)''$
$E_8(b_6)$	$D_8(a_3)$	A_5+A_1	$(A_5+A_1)'$

E₆

Character = 1
p

0:	1	1	1	1	1		1	1	1	1	1	1	1		1	1	1	1	1	1		1	1	1	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Character = 6
p

11:	1																								
10:																									
9:																									
8:	1	1																							
7:	1	1	1																						
6:																									
5:	1	1	1	1	1	1	1	1	1																
4:	1	1	1	1	1	1	1	1	1	1															
3:												1	1	1	1	1									
2:																		1	1	1	1	1	1	1	1
1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	

Character = 20
p

16:	1																						
15:	1																						
14:	1																						
13:	1																						
12:	2	1																					
11:	1	1																					
10:	2	2	1																				
9:	2	2	1																				
8:	2	2	2	1	1																		
7:	1	1	1	1	1	1	1																
6:	2	2	2	2	2	1	2	1	1	1													
5:	1	1	1	1	1	1	1	1	1														
4:	1	1	1	1	1	1	1	1	1	2	1	2	1	1	1	1							
3:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
2:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Character = 15
p

19:	1																			
18:	1																			
17:																				
16:	1																			
15:	2	1																		
14:																				
13:	1	1																		
12:	3	2	1																	
11:	1	1	1																	
10:																				
9:	2	2	1	1	1	1	1	1												
8:	1	1	1																	
7:																				
6:	1	1	1	1	1	1	1	1	1											
5:	1	1	1	1	1	1		1	1											
4:																				
3:																				

0 1 2 3 4 5 6 7 8 9 10 11 12:13:14 15 16 17 18 19 20:21

Character = 30
p

21:	1																			
20:																				
19:	1																			
18:	1																			
17:	2																			
16:	1																			
15:	3	1																		
14:	2	1																		
13:	3	2	1																	
12:	2	2																		
11:	3	3	2																	
10:	2	2	1																	
9:	3	3	3	2	2	1	1	1												
8:	1	1	1	1	1	1	1	1												
7:	2	2	2	2	2	1	2	1	1	1										
6:	1	1	1	1	1	1	1	1	1	1										
5:	1	1	1	1	1	1	1	1	2	1	2	1	1							
4:																				
3:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

0 1 2 3 4 5 6 7 8 9 10 11 12:13:14 15 16 17 18 19 20:21

Character = 15
q

20:	1																	
19:																		
18:	1																	
17:	1																	
16:	1																	
15:																		
14:	2	1																
13:	1	1																
12:	1	1	1															
11:	1	1																
10:	2	2	2	1														
9:																		
8:	1	1	1	1	1	1	1	1										
7:	1	1	1	1	1		1		1									
6:	1	1	1	1	1		1	1	1	1	1	1						
5:																		
4:	1	1	1	1	1		1	1	1	1	1	1	1		1		1	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Character = 64
p

23:	1																	
22:	1																	
21:	1																	
20:	3																	
19:	3																	
18:	3																	
17:	5	1																
16:	5	2																
15:	4	2																
14:	6	4	1															
13:	6	5	2															
12:	4	4	2															
11:	5	5	3	1	1	1												
10:	5	5	4	2	2	1	1											
9:	3	3	3	2	2	1	1											
8:	3	3	3	3	3	2	3	1	1	1								
7:	3	3	3	3	3	1	3	1	2	2								
6:	1	1	1	1	1		1		1	2		1		1				
5:	1	1	1	1	1		1	1	1	2	1	2	1	1		1	1	
4:	1	1	1	1	1		1	1	1	1	1	1	1		1	1	1	1
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Character = 60
p

```

25: 1
24: 1
23: 1
22: 2
21: 3
20: 2
19: 4
18: 5 1
17: 4 1
16: 4 2
15: 6 4 1
14: 4 4 1
13: 4 4 2
12: 5 5 3 1
11: 4 4 4 2 1
10: 2 2 2 2 2 1 1
 9: 3 3 3 3 3 2 3 1 1
 8: 2 2 2 2 2 2 2 1
 7: 1 1 1 1 1 1 1 1 1 1 1 1
 6: 1 1 1 1 1 1 1 1 1 1 1 1
 5: 1 1 1 1 1 1 1 1 1 1 1 1

```

Character = 24
p

```

24: 1
23:
22: 1
21: 1
20: 1
19: 1
18: 3 1
17: 1
16: 2 1
15: 2 1
14: 2 2 1
13: 1 1
12: 3 3 2 1 1
11: 1 1 1
10: 1 1 1 1 1
 9: 1 1 1 1 1 1 1
 8: 1 1 1 1 1 1 1 1
 7:
 6: 1 1 1 1 1 1 1 1 1 1 1 1 1 1

    0 1 2 3 4; 5 6 7 8 9 10 11 12; 13; 14 15 16

```


Character = 81
p

[illegible]

Character = 20₅

```

26: 1
25:
24: 1
23: 2
22: 1
21:
20: 3 1
19: 2 1
18:
17: 2 1
16: 3 2 1
15:
14: 1 1 1
13: 2 2 1
12: 1 1 1
11: 1
10: 1 1 1 1 1 1 1
9:
8: 1
7: 1

```

Character = 90₅

28:	1													
27:	2													
26:	1													
25:	3													
24:	4													
23:	4													
22:	5													
21:	8	1												
20:	6	1												
19:	7	2												
18:	8	4												
17:	7	5	1											
16:	6	5	1											
15:	8	7	3											
14:	5	5	3											
13:	4	4	3	1	1	1								
12:	4	4	3	2	2	2	1							
11:	3	3	3	2	2	1	1							
10:	1	1	1	1	1	1	1							
9:	2	2	2	2	2	1	2	1	1	1				
8:	1	1	1	1	1		1		1	1				
7:									1		1		1	

0 1 2 3 4: 5 6 7 8 9 10 11 12:13:14

Character = 80₅

29:	1													
28:														
27:	1													
26:	2													
25:	3													
24:	2													
23:	6													
22:	4													
21:	5													
20:	6	1												
19:	8	3												
18:	4	2												
17:	8	5	1											
16:	6	5	1											
15:	5	5	3											
14:	4	4	1											
13:	6	6	5	2	1									
12:	2	2	2	1	1									
11:	3	3	3	3	3	2	2	1						
10:	2	2	2	2	2	1	2		1					
9:	1	1	1	1	1		1		1	1				
8:														
7:	1	1	1	1	1		1	1	1	1	1	1	1	

0 1 2 3 4: 5 6 7 8 9 10 11 12:13:14

Character = 60₅

```

28: 1
27: 1
26: 2
25: 1
24: 3
23: 3
22: 4
21: 3
20: 6 1
19: 4 1
18: 4 2
17: 4 3
16: 6 5 2
15: 3 3 1
14: 4 4 3 1
13: 3 3 2 1
12: 3 3 3 2 1
11: 1 1 1 1 1 1 1
10: 2 2 2 2 2 1 2 1 1
 9: 1 1 1 1 1 1 1
 8: 1 1 1 1 1 1 1 1 1 1 1 1

    0 1 2 3 4 5 6 7 8 9 10 11

```

Character = 10₅

27:	1									
26:										
25:										
24:										
23:	1									
22:										
21:	1									
20:	1									
19:	1									
18:										
17:	1	1								
16:	1	1								
15:	1	1	1							
14:										
13:	1	1	1	1						
12:										
11:										
10:										
9:	1	1	1	1	1		1	1	1	1
	0	1	2	3	4	5	6	7	8	9 10

Character = 81'
p

30:	1									
29:	1									
28:	2									
27:	2									
26:	4									
25:	4									
24:	5									
23:	5									
22:	7	1								
21:	6	1								
20:	7	3								
19:	6	3								
18:	7	5	1							
17:	5	4	1							
16:	5	5	2							
15:	4	4	2							
14:	4	4	3	1	1					
13:	2	2	2	1	1	1				
12:	2	2	2	2	2	1	1			
11:	1	1	1	1	1	1	1			
10:	1	1	1	1	1		1		1	1
	0	1	2	3	4	5	6	7	8	9

Character = 60 p

31:	1								
30:	1								
29:	1								
28:	2								
27:	3								
26:	2								
25:	4								
24:	5								
23:	4								
22:	4								
21:	6	2							
20:	4	2							
19:	4	3							
18:	5	4	1						
17:	4	4	2						
16:	2	2	1						
15:	3	3	2	1					
14:	2	2	2	1					
13:	1	1	1	1	1				
12:	1	1	1	1	1	1	1		
11:	1	1	1	1	1		1		1
	0	1	2	3	4	5	6	7	8

Character = 24_p

30:	1								
29:									
28:	1								
27:	1								
26:	1								
25:	1								
24:	3								
23:	1								
22:	2								
21:	2	1							
20:	2	1							
19:	1	1							
18:	3	2	1						
17:	1	1							
16:	1	1	1						
15:	1	1							
14:	1	1	1						
13:									
12:	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	

Character = 64 p

32:	1						
31:	1						
30:	1						
29:	3						
28:	3						
27:	3						
26:	5						
25:	5						
24:	4						
23:	6	1					
22:	6	2					
21:	4	2					
20:	5	3					
19:	5	4	1				
18:	3	3	1				
17:	3	3	1				
16:	3	3	2				
15:	1	1	1				
14:	1	1	1	1	1	1	
13:	1	1	1	1	1	1	1
	0	1	2	3	4	5	6

Character = 15_p

31:	1					
30:	1					
29:						
28:	1					
27:	2					
26:						
25:	1					
24:	3	1				
23:	1					
22:						
21:	2	1				
20:	1	1				
19:						
18:	1	1				
17:	1	1	1			
16:						
15:						1
	0	1	2	3	4	5

Character = 30

[illegible]

Character = 15_q'

32: 1
 31:
 30: 1
 29: 1
 28: 1
 27:
 26: 2
 25: 1
 24: 1
 23: 1
 22: 2 1
 21:
 20: 1 1
 19: 1 1
 18: 1 1 1
 17:
 16: 1 1 1 1

 0 1 2 3

Character = 20_p'

34: 1
 33: 1
 32: 1
 31: 1
 30: 2
 29: 1
 28: 2
 27: 2
 26: 2
 25: 1
 24: 2 1
 23: 1 1
 22: 1 1
 21: 1 1
 20: 1 1 1

 0 1 2

Character = 6_p'

35: 1
 34:
 33:
 32: 1
 31: 1
 30:
 29: 1
 28: 1
 27:
 26:
 25: 1 1

 0 1

Character = 1_p'

36: 1

 0

E 7

10

[illegible]

1. *Introduction*

Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6

$\frac{1}{2} \log \frac{1}{2}$	0.6931
$\frac{1}{3} \log \frac{1}{3}$	1.0986
$\frac{1}{4} \log \frac{1}{4}$	1.3863
$\frac{1}{5} \log \frac{1}{5}$	1.6094
$\frac{1}{6} \log \frac{1}{6}$	1.7918
$\frac{1}{7} \log \frac{1}{7}$	1.9459
$\frac{1}{8} \log \frac{1}{8}$	2.0794
$\frac{1}{9} \log \frac{1}{9}$	2.1972
$\frac{1}{10} \log \frac{1}{10}$	2.3026
$\frac{1}{11} \log \frac{1}{11}$	2.3979
$\frac{1}{12} \log \frac{1}{12}$	2.4849
$\frac{1}{13} \log \frac{1}{13}$	2.5649
$\frac{1}{14} \log \frac{1}{14}$	2.6390
$\frac{1}{15} \log \frac{1}{15}$	2.7081
$\frac{1}{16} \log \frac{1}{16}$	2.7725
$\frac{1}{17} \log \frac{1}{17}$	2.8327
$\frac{1}{18} \log \frac{1}{18}$	2.8883
$\frac{1}{19} \log \frac{1}{19}$	2.9398
$\frac{1}{20} \log \frac{1}{20}$	2.9957
$\frac{1}{21} \log \frac{1}{21}$	3.0472
$\frac{1}{22} \log \frac{1}{22}$	3.0943
$\frac{1}{23} \log \frac{1}{23}$	3.1371
$\frac{1}{24} \log \frac{1}{24}$	3.1760
$\frac{1}{25} \log \frac{1}{25}$	3.2103
$\frac{1}{26} \log \frac{1}{26}$	3.2401
$\frac{1}{27} \log \frac{1}{27}$	3.2657
$\frac{1}{28} \log \frac{1}{28}$	3.2875
$\frac{1}{29} \log \frac{1}{29}$	3.3054
$\frac{1}{30} \log \frac{1}{30}$	3.3199
$\frac{1}{31} \log \frac{1}{31}$	3.3314
$\frac{1}{32} \log \frac{1}{32}$	3.3401
$\frac{1}{33} \log \frac{1}{33}$	3.3464
$\frac{1}{34} \log \frac{1}{34}$	3.3506
$\frac{1}{35} \log \frac{1}{35}$	3.3530
$\frac{1}{36} \log \frac{1}{36}$	3.3539
$\frac{1}{37} \log \frac{1}{37}$	3.3535
$\frac{1}{38} \log \frac{1}{38}$	3.3518
$\frac{1}{39} \log \frac{1}{39}$	3.3489
$\frac{1}{40} \log \frac{1}{40}$	3.3447
$\frac{1}{41} \log \frac{1}{41}$	3.3393
$\frac{1}{42} \log \frac{1}{42}$	3.3328
$\frac{1}{43} \log \frac{1}{43}$	3.3253
$\frac{1}{44} \log \frac{1}{44}$	3.3168
$\frac{1}{45} \log \frac{1}{45}$	3.3074
$\frac{1}{46} \log \frac{1}{46}$	3.2971
$\frac{1}{47} \log \frac{1}{47}$	3.2860
$\frac{1}{48} \log \frac{1}{48}$	3.2741
$\frac{1}{49} \log \frac{1}{49}$	3.2615
$\frac{1}{50} \log \frac{1}{50}$	3.2482
$\frac{1}{51} \log \frac{1}{51}$	3.2343
$\frac{1}{52} \log \frac{1}{52}$	3.2198
$\frac{1}{53} \log \frac{1}{53}$	3.2048
$\frac{1}{54} \log \frac{1}{54}$	3.1893
$\frac{1}{55} \log \frac{1}{55}$	3.1734
$\frac{1}{56} \log \frac{1}{56}$	3.1571
$\frac{1}{57} \log \frac{1}{57}$	3.1404
$\frac{1}{58} \log \frac{1}{58}$	3.1234
$\frac{1}{59} \log \frac{1}{59}$	3.1061
$\frac{1}{60} \log \frac{1}{60}$	3.0885
$\frac{1}{61} \log \frac{1}{61}$	3.0707
$\frac{1}{62} \log \frac{1}{62}$	3.0526
$\frac{1}{63} \log \frac{1}{63}$	3.0343
$\frac{1}{64} \log \frac{1}{64}$	3.0158
$\frac{1}{65} \log \frac{1}{65}$	2.9971
$\frac{1}{66} \log \frac{1}{66}$	2.9782
$\frac{1}{67} \log \frac{1}{67}$	2.9591
$\frac{1}{68} \log \frac{1}{68}$	2.9398
$\frac{1}{69} \log \frac{1}{69}$	2.9203
$\frac{1}{70} \log \frac{1}{70}$	2.9006
$\frac{1}{71} \log \frac{1}{71}$	2.8807
$\frac{1}{72} \log \frac{1}{72}$	2.8606
$\frac{1}{73} \log \frac{1}{73}$	2.8403
$\frac{1}{74} \log \frac{1}{74}$	2.8198
$\frac{1}{75} \log \frac{1}{75}$	2.7991
$\frac{1}{76} \log \frac{1}{76}$	2.7782
$\frac{1}{77} \log \frac{1}{77}$	2.7571
$\frac{1}{78} \log \frac{1}{78}$	2.7358
$\frac{1}{79} \log \frac{1}{79}$	2.7143
$\frac{1}{80} \log \frac{1}{80}$	2.6926
$\frac{1}{81} \log \frac{1}{81}$	2.6707
$\frac{1}{82} \log \frac{1}{82}$	2.6486
$\frac{1}{83} \log \frac{1}{83}$	2.6263
$\frac{1}{84} \log \frac{1}{84}$	2.6038
$\frac{1}{85} \log \frac{1}{85}$	2.5811
$\frac{1}{86} \log \frac{1}{86}$	2.5582
$\frac{1}{87} \log \frac{1}{87}$	2.5351
$\frac{1}{88} \log \frac{1}{88}$	2.5118
$\frac{1}{89} \log \frac{1}{89}$	2.4883
$\frac{1}{90} \log \frac{1}{90}$	2.4646
$\frac{1}{91} \log \frac{1}{91}$	2.4407
$\frac{1}{92} \log \frac{1}{92}$	2.4166
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$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x}$

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Chapter 10

[illegible]

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[illegible]

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[illegible]

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48: 6
46: 10
44: 15
42: 24
40: 30
38: 37 2
36: 45 7
34: 48 13
32: 50 23 1
30: 51 32 5 2
28: 46 36 12 4
26: 41 38 19 6 3 1
24: 35 34 24 10 8 5 1
22: 26 26 23 10 13 11 4 1
20: 19 19 18 10 15 14 8 3 1
18: 13 13 13 10 12 12 7 5 3 1
16: 7 7 7 6 7 7 3 6 6 3 1
14: 4 4 4 4 4 4 1 4 4 1 4 3 4 5 3 3 4 2 1 2 1
12: 2 2 2 2 2 2 2 2 2 2 2 3 2 2 3 2 1 2 2 1 2 1 1 1 1

```

0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17: 18: 19 20 21 22: 23 24: 25 26 27: 28 29 30 31: 32

Character = 512
a

```

51: 2
49: 4
47: 7
45: 13
43: 19
41: 26
39: 35 1
37: 41 3
35: 46 10
33: 51 19
31: 50 27 3 1
29: 48 35 8 2
27: 45 38 16 6 1
25: 37 35 22 8 5 3
23: 30 30 23 9 11 8 3 2
21: 24 24 22 12 15 13 6 4 1
19: 15 15 15 9 13 13 7 7 4 1 1 1
17: 10 10 10 8 10 10 6 8 5 7 2 3 4 2
15: 6 6 6 6 6 6 2 6 6 6 4 5 6 3 3 3 1 1
13: 2 2 2 2 2 2 2 2 2 2 2 4 2 2 4 2 2 2 2 1 1
11: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17: 18: 19 20 21 22: 23 24: 25 26 27: 28 29 30 31: 32

Character = 336

a

```

52: 1
50: 3
48: 5
46: 9
44: 14
42: 18
40: 24 1
38: 29 3
36: 31 7
34: 34 13
32: 34 18 2 1
30: 31 22 5 1
28: 29 24 9 3
26: 24 22 13 5 2 2 1
24: 18 18 13 4 5 5 3
22: 14 14 12 6 7 7 2 2 1
20: 9 9 9 5 7 7 5 3 3 2 1
18: 5 5 5 3 5 5 4 3 3 2 1 2 1
16: 3 3 3 3 3 3 2 3 3 3 1 2 4 2
14: 1 1 1 1 1 1 1 1 1 1 1 3 1 1 3 1 2 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17: 18: 19 20 21 22: 23 24: 25 26 27: 28

```

Character = 420

a

```

53: 1
51: 2
49: 5
47: 9
45: 14
43: 20
41: 27
39: 32 2
37: 38 6
35: 41 12
33: 42 19 1
31: 41 26 4 1
29: 38 29 9 3
27: 32 29 14 4 1 1 1
25: 27 26 17 6 5 4 4
23: 20 20 17 7 8 8 1 1 2
21: 14 14 13 6 10 10 6 3 3 2
19: 9 9 9 6 8 8 6 5 5 4 3 3 1
17: 5 5 5 4 5 5 3 4 4 4 1 2 4 1
15: 2 2 2 2 2 2 1 2 2 1 2 1 2 3 1 1 2 1 1
13: 1 1 1 1 1 1 1 1 1 1 1 2 1 1 2 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17: 18: 19 20 21 22: 23 24: 25 26 27: 28

```


25

Character - 405'

```

53: 1
53: 2
51: 5
49: 8
47: 14
45: 19
43: 26
41: 31 1
39: 37 4
37: 39 9
35: 41 16
33: 39 22 2 1
31: 37 27 6 2
29: 31 27 11 4
27: 26 25 15 5 3 2
25: 17 19 15 6 6 5
23: 14 14 13 6 9 8
21: 8 8 8 5 7 7
19: 5 5 5 4 5 5
17: 2 2 2 2 2 2
15: 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17: 18: 19 20 21 22: 23

```

[illegible]

Character - 216_a

54:	1																			
52:	2																			
50:	4																			
48:	6																			
46:	9																			
44:	12																			
42:	15																			
40:	18	1																		
38:	20	3																		
36:	21	6																		
34:	21	10																		
32:	20	13	2	1																
30:	18	15	5	2																
28:	15	14	7	3	1															
26:	12	12	9	4	3	1														
							1	1												
24:	9	9	8	4	5	3		2	2											
22:	6	6	6	4	5	4		2	3	2										
20:	4	4	4	3	4	4		2	3	3	1	2	1							
18:	2	2	2	2	2	2		1	2	2	1	2	1	1	1	1				
16:	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1				

0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17:18:19 20

Character - 35_a

50:	1																			
48:	1																			
46:	2																			
44:	2																			
42:	3																			
40:	3	1																		
38:	4	1																		
36:	3	1																		
34:	4	2																		
32:	3	2	1																	
30:	3	2																		
28:	2	2	1																	
26:	2	2	1																	
24:	1	1	1	1																
22:	1	1	1		1	1	1													
20:																				
18:																				
16:																				

0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16 17:18:19

Character = 280_b

55: 1
53: 3
51: 4
49: 6
47: 12
45: 15
43: 20
41: 24 1
39: 25 3
37: 28 8
35: 28 12
33: 25 16 2
31: 24 19 5 1
29: 20 16 9 3 1
27: 15 15 10 2 3 1
25: 12 12 10 4 5 3 1
23: 6 8 8 4 6 5 2 1
21: 4 4 4 2 4 4 3 2 1
19: 3 3 3 3 3 3 2 3 3 2 1
17: 1 1 1 1 1 1 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15 16

Character = 78_a

54: 1
52: 1
50: 1
48: 3
46: 3
44: 4
42: 6
40: 6
38: 6 1
36: 8 3
34: 6 3
32: 6 5 1
30: 6 5 2 1
28: 4 4 3 1 1
26: 3 3 2 1
24: 3 3 3 2 2 1 1
22: 1 1 1 1 1 1 1
20: 1 1 1 1 1 1 1 1 1
18: 1 1 1 1 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14 15

Character = 189_c

56: 1
54: 2
52: 4
50: 6
48: 9
46: 12
44: 15
42: 17 1
40: 19 3
38: 19 6
36: 19 9
34: 17 11 2 1
32: 15 12 3 1
30: 12 11 5 2
28: 9 9 6 2 1 1 1
26: 6 6 5 2 2 2 2
24: 4 4 4 2 3 3 2 1 1
22: 2 2 2 1 2 2 2 1 1 1
20: 1 1 1 1 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13 14

Character = 210_a

57: 1
55: 2
53: 4
51: 7
49: 10
47: 13
45: 17
43: 19 1
41: 21 3
39: 22 6
37: 21 9
35: 19 12 1
33: 17 13 3 1
31: 13 12 5 1
29: 10 10 6 1 1 1
27: 7 7 6 2 2 2 1
25: 4 4 4 1 3 3 2
23: 2 2 2 1 2 2 1 1 1
21: 1 1 1 1 1 1 1 1 1
0 1 2 3 4 5: 6 7 8: 9 10 11 12 13

Character = 168_a

57: 1
55: 2
53: 3
51: 6
49: 8
47: 10
45: 14
43: 15
41: 16 2
39: 18 5
37: 16 7
35: 15 10 1
33: 14 11 3 1
31: 10 9 4 1
29: 8 6 5 1 1
27: 6 6 5 2 2 1
25: 3 3 3 1 2 2 2 1
23: 2 2 2 1 2 2 2 1 1 1
21: 1 1 1 1 1 1 1 1 1 1 1

0 1 2 3 4 5 6 7 8 9 10 11 12

Character = 185_b

57: 1
55: 1
53: 2
51: 4
49: 5
47: 6
45: 9
43: 9
41: 10 1
39: 11 3
37: 10 4
35: 9 6
33: 9 7 2 1
31: 6 6 3 1
29: 5 5 3 1 1
27: 4 4 4 2 2 1 1
25: 2 2 2 1 2 1 1 1
23: 1 1 1 1 1 1 1 1
21: 1 1 1 1 1 1 1 1 1 1

0 1 2 3 4 5 6 7 8 9 10 11

Character = 189_b

58: 1
56: 2
54: 4
52: 6
50: 9
48: 12
46: 15
44: 17
42: 19 2
40: 19 4
38: 19 8
36: 17 10
34: 15 12 3 1
32: 12 11 4 1
30: 9 9 6 2 1
28: 6 6 5 1 2 1 1
26: 4 4 4 2 3 2 1
24: 2 2 2 1 2 2 1 1
22: 1 1 1 1 1 1 1 1 1

0 1 2 3 4 5 6 7 8 9 10

Character = 185_a

58: 1
56: 2
54: 3
52: 5
50: 7
48: 8
46: 10
44: 11 1
42: 11 2
40: 11 4
38: 10 5
36: 6 6 1
34: 7 6 1
32: 5 5 3 1
30: 3 3 2
28: 2 2 2 1 1 1 1
26: 1 1 1 1 1 1 1

0 1 2 3 4 5 6 7 8 9

Character = 120_g

59: 1
57: 1
55: 3
53: 5
51: 6
49: 9
47: 11
45: 11
43: 13 2
41: 13 3
39: 11 5
37: 11 7
35: 9 7 2 1
33: 6 6 2
31: 5 5 3 1
29: 3 3 3 1 1 1
27: 1 1 1 1 1 1
25: 1 1 1 1 1 1 1 1

0 1 2 3 4 5 6 7 8 9

Character = 15_g

56: 1
54: 1
52: 1
50: 1
48: 1
46: 1
44: 2
42: 1
40: 2 1
38: 1
36: 1 1
34: 1 1
32: 1 1 1 1
30: 1
28: 1 1 1 1 1 1
0 1 2 3 4 5 6 7

Character = 21_g

57: 1
55: 1
53: 1
51: 2
49: 2
47: 2
45: 3 1
43: 2
41: 2 1
39: 2 1
37: 1 1
35: 1 1
33: 1 1 1
31: 1
0 1 2 3 4 5 6

Character = 56_g

60: 1
58: 1
56: 2
54: 3
52: 4
50: 5
48: 6
46: 6
44: 6 1
42: 6 2
40: 5 3
38: 4 3
36: 3 3 1
34: 2 2 1
32: 1 1 1
30: 1 1 1 1 1
0 1 2 3 4 5 6

Character = 35_b

57: 1
57: 1
55: 2
53: 2
51: 3
49: 3
47: 4
45: 3
43: 4 1
41: 3 1
39: 3 2
37: 2 2
35: 2 2 1
33: 1 1 1
31: 1 1 1 1

0 1 2 3 4

Character = 21_b

60: 1
58: 1
56: 1
54: 2
52: 2
50: 2
48: 3
46: 2
44: 2 1
42: 2 1
40: 1 1
38: 1 1
36: 1 1 1 1

0 1 2 3

Character = 27_a

61: 1
59: 1
57: 2
55: 2
53: 3
51: 3
49: 3
47: 3
45: 3 1
43: 2 1
41: 2 2
39: 1 1
37: 1 1 1

0 1 2

Character = 7_a

62: 1
60:
58: 1
56: 1
54: 1
52: 1
50: 1
48:
46: 1 1

0 1

Character = 1_a

63: 1

0

F₈

Character = 35

46: 1
44: 1
42: 1
40: 1
38: 1
36: 2
34: 2
32: 1
30: 3
28: 2
26: 2
24: 3
22: 2
20: 2
18: 3
16: 1
14: 2
12: 2
10: 1
8: 1
6: 1
4: 1
2: 1

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58

8: 1
6: 1
4: 1
2: 1

59: 60: 61: 62: 63: 64: 65 66 67: 68 69 70 71: 72 73: 74 75: 76 77 78: 79 80 81 82: 83 84: 85 86: 87: 88 89: 90 91 92: 93: 94 95 96: 97: 98 99 100: 101 102 103: 104 105: 106 107 108: 109 110

Character = 1

Character = 56

44

-82-

59: 4 1
57: 1
55: 2 1
53: 5 2
51: 1 1
49: 4 2
47: 5 3
45: 1
43: 5 4 2
41: 4 3 1 1
39: 1 1 1 1
37: 5 4 3 1 1 2
35: 2 2 1 1 1
33: 1 1 1 1
31: 4 4 3 2 2 1 1
29: 1 1 1 1 1
27: 1 1 1 1
25: 2 2 2 1 1 1
23: 1 1 1 1
21: 1 1 1 1 1
19: 1 1 1 1 1
17: 1 1 1 1
15: 1 1 1 1
13: 1 1 1 1
11: 1 1 1 1

1

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58

59: 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98

Character = 350^x

66:111

64:11 I

101-129

60: 17 4

58: 16 4

56: 14 5

54: 23 19

52: 18 9

50: 18 11

48)-24 16

46: 18 13

44: 18 15

42: 23 19

48: 14 13

38: 16 15

36; 17 16

54: 10 10 10

32: 11 11
70: 11 11

11:30

187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

1000

24
27

2 2 177

20. 7
19. 7

71

100

100

100

67

12:

22



59:60:61:

Character = 350^x

— 20 —

64:111

62:1011

69:17 4

58: 16 4

56: 14 5

54: 23 19 1

52: 18 9 1

50: 18 11 3

48)-24 16 5

46: 18 13 5 1

44: 18 15 8 1

42: 23 19 11 4

48: 14 13 8 15

38: 16 15 12 3

36: 17 16 12 7

9	6	01	01	55
9	6	01	01	55

9 01 11 11 :25

6
01
11
11
30:

187

	L	U
-	L	U
-	L	U
-	L	U
2	2	2

27	24
----	----

2	2	2	2	100
7	7	7	7	177

20. 7 7 7 7
19. 7 7 7 7

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14: 1 1

12: 1 1

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59:60:61:62:63:64:65

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76: 175 5
74: 208 12
72: 222 19
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46: 208 196 141 55 47 27 6 2 1
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16: 1

59: 60: 61: 62: 63: 64: 65

Character = 420
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[illegible]

Character = 2100

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[illegible]

Character = 5698' z

[illegible]

191 = 1234567

[illegible]

Character = 4200²

[illegible]

Case - 2002-07

[illegible]

Character = 4280' x

[illegible]

Character = 200^x

[illegible]

Character = 4096,
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83: 127 1
81: 156 5
79: 170 8
77: 182 15
75: 200 20
73: 213 39
71: 219 56
69: 236 78 1
67: 226 92 3
65: 223 111 9
63: 227 131 18
61: 204 135 28
59: 194 144 45 2 1 1
57: 185 148 59 6 3 2
55: 155 135 68 11 8 7
53: 142 130 80 18 13 12 1
51: 126 119 82 28 22 14 3 1 3
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The first column is not palindromic.
The remaining coefficients are:
109: 1
107: 1
105: 3
103: 7
101: 9
99: 17
97: 25
95: 31
93: 47
91: 61
89: 72
87: 96
85: 113

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

Character = 4996 X

The first column is not palindromic.
The remaining coefficients are:

84:	126	1
82:	142	2
80:	155	5
78:	185	13
76:	194	20
74:	204	32
72:	227	51
70:	233	64
68:	226	84
66:	236	107
64:	219	117
62:	213	133
60:	200	145
58:	182	141
56:	162	141
54:	142	141
52:	122	141
50:	102	141
48:	82	141
46:	62	141
44:	42	141
42:	22	141
40:	2	141
38:		141
36:		141
34:		141
32:		141
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22:		141
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14:		141
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10:		141
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4:		141
2:		141
0:		141

Character = 700' XX

[illegible]

[illegible][illegible]

Character = 972' x

[illegible]

Character = 350^x

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82:	16	1
80:	14	1
78:	23	4
76:	18	3
74:	18	5
72)-	24	8
70:	18	7
68:	18	9
66:	23	13
64:	14	9
62:	16	12
60:	17	13
58:	10	9
56:	11	10
54:	11	10
52:	5	5
50:	7	7
48:	5	5
46:	2	2
44:	3	3
42:	2	2
40:		
38:	1	1
36:		
34:		
32:		
	0	1
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Character = 1575^x

86:	60	1
84:	61	1
82:	71	4
80:	82	7
78:	77	9
76:	89	17
74:	92	23
72:	85	27
70:	92	38
68:	89	43
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64:	82	54
62:	71	52
60:	61	58
58:	60	52
56:	48	44
54:	38	37
52:	37	36
50:	25	25
48:	20	20
46:	17	17
44:	10	10
42:	7	7
40:	6	6
38:	2	2
36:	2	2
34:	1	1
32:		

0 1 2 3 4: 5 6 7: 8 9 10 11 12: 13 14 15 16: 17: 18 19 20 21 22: 23: 24

84:	62	2
82:	61	2
80:	66	5
78:	77	10
76:	76	13
74:	74	18
72:-	86	28
70:	74	29
68:	76	38
66:	77	44
64:	66	44
62:	61	46
60:	62	50
58:	45	40
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48:	21	21
46:	11	11
44:	12	12
42:	7	7
40:	4	4
38:	3	3
36:	2	2
34:		
32:	1	1
	0	1

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[illegible]

Character = 56'
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81: 1 1
79: 4 1
77: 5 1
75: 5 2
73: 5 2
71: 4 2
69: 1 1
67: 5 3 1
65: 2 1
63: 1 1
61: 4 3 1
59: 1 1
57: 1 1 1
55: 2 2 1
53: 1 1 1
51: 1 1 1
49: 1 1 1
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45: 1
43: 1
41: 1
39: 1
37: 1

0 1 2 3 4: 5 6 7: 8 9 10 11 12: 13 14 15 16: 17: 18

Character = 1008'
z

87: 44 1
85: 45 1
83: 48 3
81: 57 6
79: 55 8
77: 56 12
75: 62 18
73: 56 20
71: 55 25 1
69: 57 30 2
67: 48 30 4
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63: 44 34 10
61: 34 29 12 1
59: 31 28 15 1
57: 28 26 16 4
55: 19 19 14 4
53: 17 17 14 5
51: 14 14 12 7
49: 8 8 8 5
47: 7 7 7 5
45: 5 5 5 5
43: 2 2 2 2
41: 2 2 2 2
39: 1 1 1 1
37: 1

0 1 2 3 4: 5 6 7: 8 9 10 11 12: 13 14 15 16: 17: 18

Character = 1000' Z

85:	64	2
83:	72	4
81:	70	6
79:	79	12
77:	82	17
75:-	76	21
73:	82	30
71:	79	35
69:	70	38
67:	72	45
65:	64	45
63:	54	43
61:	53	45
59:	43	39
57:	34	33
55:	32	31
53:	23	23
51:	17	17
49:	15	15
47:	9	9
45:	6	6
43:	5	5
41:	2	2
39:	1	1
37:	1	1
	0	1
		2

Character = 1344'x

[illegible]

Character = 780' X

86:	35	1
84:	37	2
82:	39	4
80:	39	6
78:-	42	10
76:	39	12
74:	39	16
72:	37	18
70:	35	21
68:	31	21
66:	30	23
64:	24	20
62:	22	20
60:	18	17
58:	15	15
56:	11	11
54:	10	10
52:	6	6
50:	5	5
48:	3	3
46:	2	2
44:	1	1
42:	1	1

Order	Time	Distance	Time	Distance	Time	Distance
0	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
7	8	9	10	11	12	13
8	9	10	11	12	13	14
9	10	11	12	13	14	15

Character = 400²

85:	21	1
83:	23	2
81:	20	2
79:	24	5
-		
77:	24	6
75:	20	7
73:	23	10
71:	21	11
69:	17	11
67:	19	14
65:	15	12
63:	12	11
61:	13	12
59:	9	9
57:	7	7
55:	7	7
53:	4	4
51:	3	3
49:	3	3
47:	1	1
45:	1	1
43:	1	1
	0	1
	2	3
	4	5
	6	6

0 1 2 3 4 5 6 7 8 9 10 11

Character = 160²

89:	11	1
87:	7	
85:	11	2
83:	11	2
81:	7	2
79:	11	4
77:	9	4
75:	6	3
73:	9	6
71:	6	4
69:	4	4
67:	6	5
65:	3	3
63:	2	2
61:	3	3
59:	1	1
57:	1	1
55:	1	1
53:		

0 1 2 3 4 5 6 7 8

Character = 210^x

88:	13	1	
86:	13	1	
84:-	12	2	
82:	13	3	
80:	13	4	
78:	10	4	
76:	12	6	
74:	18	6	
72:	8	6	1
70:	8	6	1
68:	7	6	3
66:	4	4	2
64:	5	5	2
62:	3	3	2
60:	2	2	2
58:	2	3	2
56:	1	1	1
54:			
52:	1	1	1
	6	1	2

Character = 84'
x

116:	1	
114:	1	
112:	1	
110:	2	
108:	2	
106:	2	
104:	4	
102:	3	
100:	4	
98:	5	
96:	4	
94:	5	
92:	6	
90:	4	
88:	6	1
86:	5	1
84:	4	1
82:	5	2
80:	4	2
78:	3	2
76:	4	3
74:	2	2
72:	2	2
70:	2	2
68:	1	1
66:	1	1
64:	1	1
	0	1
	2	3

Character = 35'
x

118:	1	
116:		
114:	1	
112:	1	
110:	1	
108:	2	
106:	2	
104:	1	
102:	3	
100:	2	
98:	2	
96:	3	
94:	2	
92:	2	
90:	3	1
88:	1	
86:	2	1
84:	2	1
82:	1	1
80:	1	1
78:	1	1
76:		
74:	1	1
	0	1
	2	

Character = 8'
z

119:	1	
117:		
115:		
113:	1	
111:		
109:	1	
107:	1	
105:		
103:	1	
101:	1	
99:		
97:	1	
95:		
93:		
91:	1	1
	0	1

Character = 1'
x

120:	1	
	0	

II. Inclusions and branching of closures of unipotent classes

Let $u, v \in G$ be unipotent, $\phi \in A(u)^\wedge$ and $\rho = \rho_{v,1} \in W^\wedge$. Then u is in the closure X of the class of v if and only if $n_{u,1,\beta(v),\rho} \neq 0$. Moreover, $n_{u,\phi,\beta(v),\rho}$ is the multiplicity of ϕ in the permutation representation of $A(u)$ on the set of branches of X at u .

We give here the integers $n_{u,\phi,\beta(v),\rho}$ ($\rho = \rho_{v,1}$). The pair (u,ϕ) is labelled as before and v is assigned the number corresponding to $(v,1)$.

If r,s correspond respectively to $(u,\phi), (v,1)$, let $n_{r,s} = n_{u,\phi,\beta(v),\rho}$. Then $n_{r,s} = 0$ if $r > s$. Moreover:

for E_7 : $n_{r,s} = n_{r,31}$ if $r \leq 20$ and $s \geq 31$;

for E_8 :

$$n_{r,s} = \begin{cases} n_{r,36} & \text{if } r \leq 31, r \neq 20 \text{ and } s \geq 36 \\ n_{r,82} & \text{if } r = 20 \text{ or } 32 \leq r \leq 66 \text{ and } s \geq 82 \text{ or } s = 77. \end{cases}$$

The remaining values are given in the table.

To save space, the line and the column 21 in the table for E_7 have been displaced. For E_8 the lines and the columns 20 and 77 have been displaced.

Inclusions and branching of closures of unipotent classes in E_8 .

[illegible]

112 111 110 108 107 105 103 102 100 99 96 95 92 91 89 86 84 77 82 81 80 78 75 73 71 70 69 67

82: 1
81: 1
80: 1
79:
78: 1 1 1
76:
75: 1 1 1
74:
73: 1 1 1 1
72:
71: 1 1 1 1 1 1 1
70: 1 1 1 1 1 1
69: 1 1 1 1 1 1 1 1 1
68: 1 1 1
67: 1 1 1 1 1 1 1
66: 1 1 1 1 1 1 1 1 1
65:
64:
63:
62:
61: 1
60: 1 1 1 1 1
59: 1 1 1 1 1 1 1 1 1 1
58:
57: 1 1 1 1 2
56: 1 1 2 1 2 1 2 1 2 3 1 1 1
55: 1 1 1 1 2 1 1
54: 1 1 2 1 2 1 2 1 2 3 1 1 1 1
53: 1 1 1 1 1
52: 1 1 2 1 2 1 2 1 2 3 1 1 1 1
51: 1 1 2 1 2 1 2 1 2 3 2 1 2 1 1 1
50: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1
49: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
48: 1 1 1 1 1 1 1 1 1 1 1
47: 1 1 1 1 1
46: 1 1 2 1 2 1 2 1 2 3 1 1 1 1 1 1
45:
44: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1
43: 1 1 2 1 2 1 2 1 2 3 2 1 2 1 1 1
42: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1
41: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
40: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
39:
38: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
37:
36: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1
35:
34: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
33: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
32: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1
20: 1 1 2 1 2 1 2 1 2 2 1 1 1 1 1 1

82 81 80 78 75 73 71 70 69 67 66 59 56 54 52 51 50 49 48 46 44 43 42 41 40 38 36 34 33 32 20

Inclusions and branching of unipotent classes in E_7 .[illegible]

59 58 57 56 54 53 51 49 48 47 46 45 42 41 39 38 37 36 34 33 31 30 29 21 27 26 24 22

```

31: 1
30: 1
29: 1
21: 1 1
28:
27: 1 1 1
26: 1 1 1
25:
24: 1 1 1 1 1 1
23: 1 1 1
22: 1 1 1 1 1 1
20: 1 1 1 1 1 1 1
19:
18: 1 1 1 1
17: 1 1 1 1 1 1 1
16: 1 2 1 1 2 1 2 2 1 1 1
15: 1 1 1 1 1 1 1 1 1 1 1
14: 1 1 1 1 1 1 1 1 1
13: 1 2 1 1 2 1 2 2 1 1 1 1 1
12: 1 1 1 1 1 1 1 1 1 1 1 1 1
11: 1 1 1 1 1 1 1 1 1 1 1 1 1
10: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
9:
8: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
7: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
6:
5: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
4: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
3: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

31 30 29 21 27 26 24 22 20 17 16 15 14 13 12 11 10 8 7 5 4 3 2 1 0